

Research Article

## Relationship between Performance Intelligence Quotient and Physical Characteristics

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### Abstract:

Generally, it may be expected that physical characteristics such as brain size, height, weight, gender and body mass index (BMI) can be associated with the performance intelligence quotient (PIQ) score. The current report examines the relationship between PIQ and physical characteristics such as brain size, height, weight, gender and BMI based on a real data set. It is derived herein that PIQ is non-constant variance random variable, and its mean is positively associated with brain size ( $P=0.0002$ ) and negatively associated with height ( $P=0.0046$ ). Variance of PIQ is negatively partially associated with brain size ( $P=0.0903$ ). It is also independent of weight, BMI and gender. PIQ is higher for the individuals with larger brain size, shorter height and irrespective of gender, body weight and BMI.

**Keywords:** Body Mass Index; Brain Size; Gamma & Log-Normal Models; Intelligence Quotient; Joint Generalized Linear Models.

### Introduction:

During the nineteenth and early twentieth centuries, the association between general mental ability (GMA) and whole brain size was almost universally accredited (Broca, 1873; Darwin, 1871; Morton, 1849; Topinard, 1878). Relationship between GMA and brain size has been studied in many review articles by Rushton and Ankney (1995, 1996, 2007, 2009). These cover many important findings that are reported in most of the earlier published articles. The famous neurologist Paul Broca (1824–1880) weighed internal and external skull dimensions and measured wet brains at autopsy and found that mature adults averaged a bigger brain than either very elderly, or the children, eminent persons averaged a bigger brain than the less eminent

and skilled workers averaged a bigger brain than the unskilled (Broca, 1873). Charles Darwin (1871) mentioned Broca's studies in his book entitled- The Descent of Man to confirm his theory of evolution. Sir Francis Galton (1888), first quantified the relation between GMA and the brain size in living individuals, and concluded that men who received high honors degrees had a brain size 2%–5% larger than those who did not. Karl Pearson (1906) analyzed Galton's data using the simple correlation coefficient ( $r$ ) and observed that the correlation coefficient value between GMA and brain size is  $r = 0.11$ , which is not statistically significant. Therefore, Karl Pearson analysis partially supported Galton's study. Spearman (1904, 1927) obtained the various

GMA items, and found positive correlation of each subset, and also observed a general factor of intelligence. National Collaborative Perinatal Project (Broman et al., 1975, 1987) data were recorded separately by gender, and correlation for body size were not included. Rushton and Ankney (2009) discussed the results of 28 studies that adopted brain imaging techniques such as computed tomography (CT) and magnetic resonance imaging (MRI) in a total of 1,389 normal subjects. The correlations between brain size and GMA range from 0.04 to 0.69.

Average brain size difference due to sex difference was not considered in the study by Broca (1873). It is frequently claimed, however, that this difference evaporates when corrections are measured for age or body size of people sampled (Gould, 1981, 1996). However, Ankney (1992) described that the gender difference in brain size remains after corrections for body size in a similarly aged women and men sample. This result was supported by Gur et al., (1991) and Willerman et al., (1991). From the review article by Rushton and Ankney (2009), it is concluded that brain size is positively correlated with intelligence, while GMA and brain size are correlated with gender, socioeconomic position, age, and population group differences. Note that for multivariate data, simple nonzero and zero correlations do not prove cause and effect, while partial nonzero correlations do provide support. All the earlier studies are based on simple correlation and usual multiple regression that invites doubts and debates. In addition, physiological data are always heteroscedastic, so usual multiple regressions is not appropriate (shown in the background section). The current paper is organized as follows. The following section reveals the background & material of the study, and the subsequent sections reveals respectively methods, results, and discussion and conclusion. Both the derived gamma and lognormal models can predict the mean PIQ.

### Background & Materials:

Willerman et al., (1991) studied PIQ based on a data set of 40 individuals. The researchers adopted MRI to measure the brain size of the individuals, and considered subjects body size also. They performed their study at a large southwestern university. The researchers selected a random sample of 40 right-handed Anglo introductory psychology students who had reported no history of unconsciousness, alcoholism, epilepsy, brain damage, or heart disease. These individuals were selected from a larger pool of introductory psychology students with total Scholastic Aptitude Test Scores lower than 940, or higher than 1350. These subjects had accepted to satisfy a course requirement by accommodating the administration of four subtests (Similarities, Vocabulary, Picture Completion, and Block Design) of the Wechsler (1981) Adult Intelligence Scale-Revised. Based on the University's research review board prior approval, selected students MRI were required to receive prorated full-scale IQs of less than 103, or greater than 130. Study subjects were equally divided by gender and IQ classification. Silverman et al., (1991) collected the data from the selected 40 subject on seven study variables such as gender (male or female), full scale IQ (FSIQ) scores based on the four Wechsler (1981) subtests, verbal IQ (VIQ) scores based on the four Wechsler (1981) subtests, performance IQ (PIQ) scores based on the four Wechsler (1981) subtests, body weight (Weight) in pounds, height (Height) in inches, total pixel count from the 18 MRI (MRI\_Count) scans. The data set is given in Willerman et al., (1991). Based on the data, we have also added one more variable known as body mass index (BMI) which is defined as  $BMI = \text{Weight}(\text{kg}) / \text{Height}(\text{m}^2)$ . For ready reference, the data set is reproduced in Table 1.

**Table 1: Intelligence data along with BMI and estimated PIQ values**

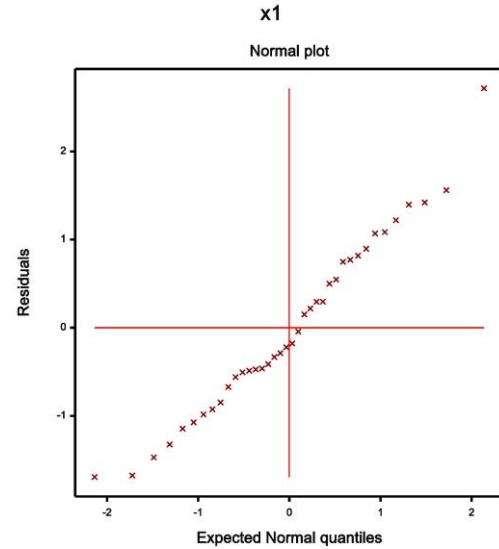
Gender	FSIQ	VIQ	PIQ	Weight	Height	MRI_Count	bmi	Esti. PIQ
Female	133	132	124	118	64.5	816932	19.93967	101.5681
Male	139	123	150	143	73.3	1038437	18.71041	119.8307
Male	133	129	128	172	68.8	965353	25.54506	117.9169
Female	137	132	134	147	65	951545	24.45941	126.1599
Female	99	90	110	146	69	928799	21.55808	110.2731
Female	138	136	131	138	64.5	991305	23.31927	136.6209
Female	92	90	98	175	66	854258	28.24265	104.4084
Male	89	93	84	134	66.3	904858	21.43054	112.9709
Male	133	114	147	172	68.8	955466	25.54506	115.9489
Female	132	129	124	118	64.5	833868	19.93967	104.5463
Male	141	150	128	151	70	1079549	21.66388	139.094
Male	135	129	124	155	69	924059	22.887	109.3955
Female	140	120	147	155	70.5	856472	21.92344	94.07102
Female	96	100	90	146	66	878897	23.56244	108.8673
Female	83	71	96	135	68	865363	20.52444	101.4108
Female	132	132	120	127	68.5	852244	19.02733	97.97771
Male	100	96	102	178	73.5	945088	23.16331	101.7654
Female	101	112	84	136	66.3	808020	21.7504	95.81316
Male	80	77	86	180	70	889083	25.82449	100.632
Male	97	107	84	186	76.5	905940	22.3432	88.59098
Female	135	129	134	122	62	790619	22.31165	103.1331
Male	139	145	128	132	68	955003	20.06834	118.0963
Female	91	86	102	114	63	831772	20.19199	107.9923
Male	141	145	131	171	72	935494	23.18924	103.788
Female	85	90	84	140	68	798612	21.2846	90.52445
Male	103	96	110	187	77	1062462	22.17254	114.2341
Female	77	83	72	106	63	793549	18.77501	101.1849
Female	130	126	124	159	66.5	866662	25.27605	105.3607
Female	133	126	132	127	62.5	857782	22.85594	114.2353
Male	144	145	137	191	67	949589	29.91156	119.8595
Male	103	96	110	192	75.5	997925	23.67896	106.1051
Male	90	96	86	181	69	879987	26.72611	101.494
Female	83	90	81	143	66.5	834344	22.73255	99.71439
Female	133	129	128	153	66.5	948066	24.32223	120.9975
Male	140	150	124	144	70.5	949395	20.36759	110.1651
Female	88	86	94	139	64.5	893983	23.48825	115.7925
Male	81	90	74	148	74	930016	19	98.00319
Male	89	91	89	179	75.5	935863	22.0757	95.49081

0.51 and  $r = 0.65$ , for women  $r = 0.33$  and  $r = 0.35$ , Willerman et al., (1991) reported simple correlation and for both gender together  $r = 0.51$ . For deriving between PIQ and brain size before and after the relationship of PIQ, multiple regression line can controlling body size, respectively as for men  $r =$  give misleading results which is clear from the multiple correlation  $R^2 = 0.2949$  and adjusted

$R^2=0.2327$  (Willerman et al., 1991). Response PIQ is a non-constant variance random variable, so usual multiple regression line can give misleading results. For ready reference, usual multiple regression fit is shown in Table 2. Figure 1(a) presents the absolute residuals plot against the fitted values, which is decreasing (a funnel shape), concluding that variance is non-constant. Figure 1(b) displays the normal probability plot for the mean model (Table 2), which indicates that there is a gap in the fitting. Therefore, both the figures 1(a) and 1(b) indicate lack of fit. In addition, from Table 2, estimated variance is  $\exp(5.971) = 391.8974$ , which is very large. Usual multiple regression line of the estimated PIQ is as follows.  
 Estimated PIQ = 111.35 + 2.06 Brain - 2.73 Height +0.001 Weight.

**Table 2: Multiple Regression Model Fitting Of PIQ With Normal Distribution**

Model	Civariate	Normal fit			
		estimate	s.e.	t-value	P-value
Mean	Constant	111.35	62.97	1.768	0.0860
	Brain size	2.06	0.56	3.657	0.0009
	Height	-2.73	1.23	-2.222	0.0330
	Weight	0.00	0.20	0.003	0.9976
Dispersion	Constant	5.971	0.2425	24.62	<0.001

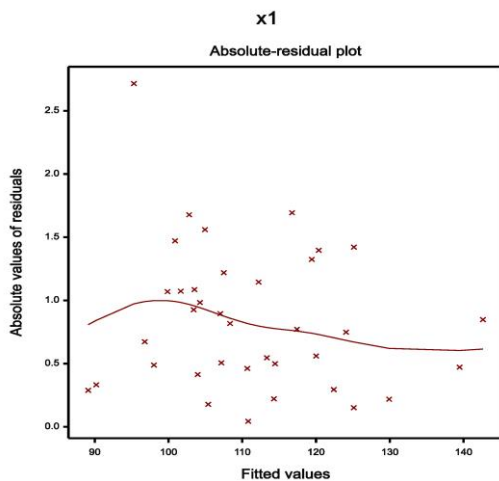


**Figure 1:** For the Normal fitted models of PIQ (Table 2), the (a) absolute student residuals plot with respect to the fitted values, and (b) the normal probability plot for the mean model.

The figures 1(a) and 1(b) show that response PIQ variance is non-constant, and the response distribution is non-normal. So usual multiple regression can give misleading results. Under that case, generally, transformation on the response variable is used to stabilize the variance, but variance may not be stabilized always (Myers et al., 2002). The response PIQ is a positive, continuous, and non-constant variance random variable. Generally, a positive, continuous, and constant variance random variable can be analyzed either by a lognormal or gamma model (Firth, 1988). If the variance is non-constant, it can be analyzed by joint generalized linear models (JGLM) adopting lognormal and gamma models (Das and Lee 2009). JGLMs is clearly given in the book by Lee et al. (2017). For ready reference it is shortly described in the method section.

**Statistical Methods**

**Lognormal Jglms:** For the positive continuous response (PIQ) random variable ( $PIQ = y_i$ 's) with non-constant variance ( $\sigma_i^2$ ), and mean  $\mu_i = E(y_i)$ , satisfying  $Var(y_i) = \sigma_i^2 \mu_i^2 = \sigma_i^2 V(\mu_i)$  say, while  $V(\cdot)$  is called as variance function, the log transformation  $z_i = \log(PIQ=y_i)$  is generally



considered to stabilize the variance  $\text{Var}(z_i) \approx \sigma_i^2$ , but the variance may not be stabilized always (Myers et al., 2002). For obtaining an advanced model, JGLMs for the mean and dispersion are in practice used. Considering the response PIQ distribution as lognormal, the JGLM of the mean and dispersion model (response  $\text{PIQ} = y_i$ , with  $z_i = \log(\text{PIQ} = y_i)$ ) are displayed by  $E(z_i) = \mu$ ,  $\text{Var}(z_i) = \sigma_{z_i}^2$ , and  $\log(\sigma_{z_i}^2) = g_i^t \gamma$  where  $x_i^t$  and  $g_i^t$  are the vectors of independent variables associated with the regression coefficients  $\beta$  and  $\gamma$ , respectively.

**Gamma Jglms:** For the response ( $\text{PIQ} = y_i$ 's) as above, its variance has two parts such that  $\sigma_i^2$  (does not depend on mean changes) and  $V(\mu_i)$  (depends on the mean changes), while  $V(\cdot)$  is called as the variance function, which recognizes the GLM family distribution. For instance, if  $V(\mu) = 1$ , it is Normal, and it is gamma, or Poisson according as  $V(\mu) = \mu^2$ , or  $V(\mu) = \mu$  etc. Gamma JGLMs mean & dispersion models for PIQ are represented by

$$\eta_i = g(\mu_i) = x_i^t \beta \text{ and } \varepsilon_i = h(\sigma_i^2) = w_i^t \gamma,$$

where  $g(\cdot)$  &  $h(\cdot)$  are the GLM link functions for the mean & dispersion linear predictors respectively, and  $x_i^t$ ,  $w_i^t$  are the vectors of explanatory variables, related with the mean and

dispersion parameters respectively. Maximum likelihood (ML) method is used to estimate mean parameters, while the restricted ML (REML) method is adopted to estimate dispersion parameters (Lee et al., 2017).

**Statistical & Graphical Analysis:**

The response PIQ is modeled by JGLMs with both lognormal & gamma distributions. Here PIQ is treated as the response, and the others brain size, gender, height, weight, BMI are treated as independent variables. Here it is shown in Figure 1 that the variance of the response PIQ is heteroscedastic, so the best JGLMs model has been accepted based on the lowest Akaike information criterion (AIC) value (within each class) that minimizes both the squared error loss and predicted additive errors (Hastie et al. 2009, p. 203-204). Based on the AIC criterion, both the JGLMs gamma (AIC=328.435) and lognormal (AIC=328.1) and fits give similar results as the AIC difference is less than one, which is insignificant. The final PIQ gamma and lognormal JGLMs analysis outcomes are displayed in Table 3.

**Table 3: Final Joint Lognormal And Gamma Model Fitting Of PIQ**

Model	Covariate	Gamma fit				Log-normal fit			
		estimate	s.e.	t-value	P-value	estimate	s.e.	t-value	P-value
Mean	constant	4.780	0.4700	10.169	<0.0001	4.716	0.4700	10.034	<0.0001
	Brain size	0.017	0.0043	4.088	0.0002	0.018	0.0043	4.303	0.0001
	Height	-0.024	0.0080	-3.031	0.0046	-0.025	0.0080	-3.096	0.0038
Dispersion	Constant	1.572	2.920	0.541	0.5919	1.4765	2.933	0.503	0.6181
	Brain size	-0.0561	0.032	-1.742	0.0903	-0.0550	0.032	-1.699	0.0981
AIC		328.435				328.1			

The derived PIQ (Table 3) probabilistic model is a data developed model that is tested adopting model diagnostic tools in Figure 2. For the joint gamma fitted PIQ models (Table 3), graphical diagnostic analysis is displayed in Figure 2. Figure 2(a) presents the absolute residuals for the fitted PIQ against the fitted values that is nearly flat linear straight line, concluding that variance is constant with the running means. In addition, funnel shape scattered plots is randomly distributed in Figure 2(a). Figure 2(b) represents the normal probability plot for the fitted PIQ mean model (Table 3), which does not show any lack of fit. Figure 2 does not present any discrepancy in the fitted PIQ model (Table 2) that supports that the gamma fitted PIQ model (Table 3) is an approximate of its true model.

**Figure 2:** For the joint gamma fitted models of PIQ (Table 2), the (a) absolute student residuals plot with respect to the fitted values, and (b) the normal probability plot for the mean model.

**Results:**

From Table 3, it is shown that mean PIQ is positively associated with brain size (P=0.0002) and it is negatively associated with height (P=0.0046). Variance of PIQ is negatively partially associated with brain size (P=0.0903).

Gamma fitted PIQ mean ( $\hat{\mu}$ ) model (Table 2) is

$$\hat{\mu} = \exp.( 4.780 + 0.017 \text{ Brain} -0.024 \text{ Height}),$$

and the gamma fitted PIQ dispersion ( $\hat{\sigma}^2$ ) model is

$$(\hat{\sigma}^2) \text{ model is}$$

$$\hat{\sigma}^2 = \exp.( 1.5792 -0.0561 \text{ Brain}).$$

Lognormal fitted PIQ mean ( $\log\text{PIQ}=\hat{\mu}$ ) model (Table 2) is

$$\log\text{PIQ}=\hat{\mu} = 4.716+ 0.018 \text{ Brain} - 0.025 \text{ Height},$$

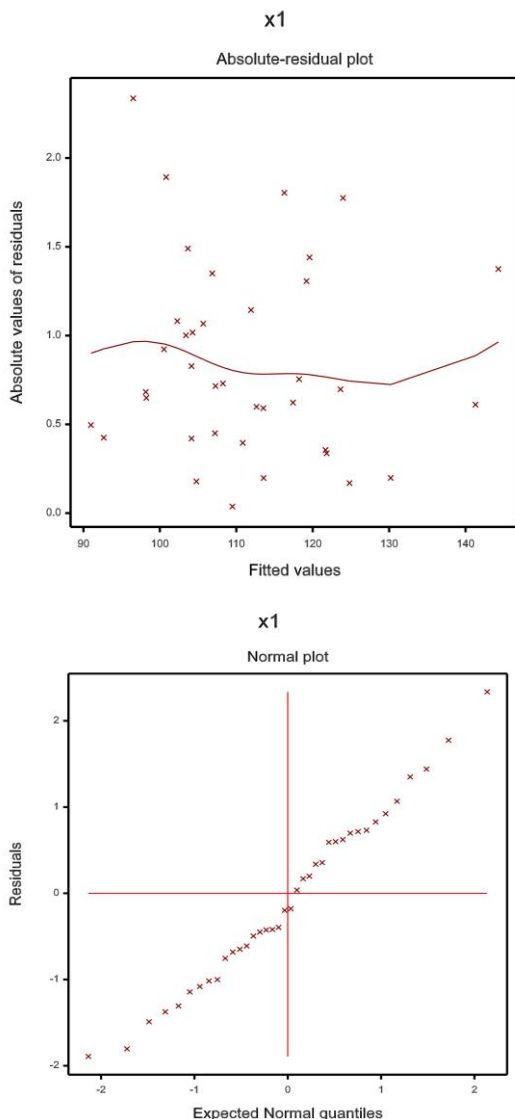
and the gamma fitted PIQ dispersion ( $\hat{\sigma}^2$ ) model is

$$\hat{\sigma}^2 = \exp.( 1.4765 -0.0550 \text{ Brain}).$$

**Discussion & Conclusions:**

The IQ data set is always a multivariate form. In case of a multivariate data set, the association between two variables can only be identified by suitable modeling of the response along with the all questionable explanatory variables. Note that IQ data set is physiological data, so variance is always non-constant due to heterogeneity of the sample subjects. So, using only JGLMs, appropriate associations can be identified. Best of our knowledge, JGLMs are not used in earlier IQ data analysis. Hope that JGLMs can give many interesting results of the previously reported IQ data analysis.

Table 3 presents the summarized PIQ data analysis outcomes. It is derived herein that mean PIQ is positively associated with brain size (P=0.0002), concluding that PIQ is always higher for the individuals with larger brain size than



smaller. This is reported in all previous research articles (Rushton and Ankney, 2009). Also mean PIQ is negatively associated with height (P=0.0046), implying that shorter individuals have higher PIQ than taller. This is not properly reported in many research articles (Rushton and Ankney, 2009). Variance of PIQ is negatively partially associated with brain size (P=0.0903), indicating that scatteredness of PIQ is smaller for the individuals having larger brain size. In other words, most of the individuals having larger brain size must have higher PIQ level. This is not reported in any previous research articles (Rushton and Ankney, 2009). Some research articles have reported that PIQ is associated with body weight and gender (Ankney, 1992; Rushton and Ankney, 2009). In Table 4, it is shown herein that PIQ is not associated with body weight, BIM and gender. The derived estimates have smaller standard error (Table 3 & 4), concluding that estimates are stable. The present accepted mean and dispersion models have been selected based on graphical diagnosis, smallest standard errors of

the estimates, smallest AIC value, and comparison of both lognormal and gamma distributions. Estimated variance is  $\hat{\sigma}^2 = \exp.(1.5792 - 0.0561 \text{ Brain})$ , which lies between 0.0116 (for the largest brain size 107.95 in the considered data set) and 0.0586 (for the smallest brain size 79.06 in the considered data set). The present outcomes satisfy the most accepted results. In addition, it gives some new results, and it removes many contradictory outcomes. The estimated PIQ values are given in Table 1, which reveal that estimates are very close to observed values. PIQ is higher for the individuals with larger brain size, shorter height and irrespective of gender, body weight and BMI.

**Table 4: Joint Lognormal And Gamma Model Fitting Of PIQ With BMI And Gender**

Model	Covariate	Gamma fit				Log-normal fit			
		estimate	s.e.	t-value	P-value	estimate	s.e.	t-value	P-value
	Constant	4.630	0.889	5.21	<0.001	4.473	0.888	5.04	<0.001
	Brain size	0.018	0.005	3.75	<0.001	0.019	0.005	3.95	<0.001
	Height	-0.023	0.010	-2.23	0.033	-0.023	0.010	-2.22	0.034
	BMI	0.001	0.012	0.11	0.916	0.003	0.012	0.25	0.806
	Gender	0.016	0.088	0.18	0.855	0.023	0.088	0.26	0.797
Disper-sion	Constant	1.231	3.084	0.40	0.692	1.086	3.071	0.35	0.726
	Brain size	-0.052	0.034	-1.52	0.139	-0.050	0.034	1.47	0.150
AIC		328.7				328.3			

**Conflict Of Interest:**

The authors confirm that this article content has no conflict of interest.

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