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Research Article

The Students' Thinking Process on Mathematics Problem Solving Through Scaffolding

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Abstract: The research aims to describe the students' thinking process on mathematics problem solving through scaffolding. The research participants were 6 students of grade 9 on junior high school in Banyuwangi Regency, East Java, Indonesia, whose determination is based on their level of mathematical ability. Question used as a research instrument was essay questions that must be done based on Polya's four stages of mathematics problem solving. Participants were given the opportunity to do reflection on what they had done. The researcher invited them for a discussion and guided them to fix the solution (*scaffolding*). The result show that participants' thinking process in problem solving before *scaffolding* there is happen imperfection in the stage of using mathematic concept that have been studied previously. Generally, by giving various *scaffolding* based on the participants' need, participants' thinking process improved and completed four stages of problem solving that were studied in this research. Moreover, by giving this *scaffolding*, students' ability on mathematics problem solving improved.

Keywords: Thinking process, Mathematic Problem Solving, Scaffolding.

INTRODUCTION

In problem solving learning, it is possible for a student to obtain experience using the knowledge and the skill s(he) has had to solve problem s(he) faces. In fact, students generally have understood the concept of materials they have studied, but they have not been able to connect and use the concept to solve the problems they face. A research conducted by Sujiati (2011) showed that when junior high school students with low math skills solve the problem, occur error or imperfection of thinking process so that there is mistake in the answer. Therefore, knowing the thinking process of students in solving a mathematical problem is necessary. By knowing the thinking process of students in problem solving, the teacher can track where and what type of mistakes made by students. Furthermore, mistakes made by students can be used as a source of learning information and understanding for students. Moreover, from this mistake, the teacher can immediately fix the student's cognitive scheme/structure.

In relation to the above statement, and based on the experience of the researcher as a teacher on junior high school in Banyuwangi Regency, East Java, Indonesia, in general the students have understood the basic concept of congruence and 2-dimensionals geometric shapes, but they can not use and connect the concept to solve the problem they faces. The mistake made by students indicate on the imperfection of students' thinking process in the four stages of the problem solving proposed by Polya (1973), those are: (1) understand the problem; (2) device a plan; (3) carry out the plan; and (4) look back, as well as in the four dimensions of the basic framework in problem solving proposed by Wu (2006), those are: (1) reading/extracting all information from the question; (2) real-life and common sense approach to solving problems; (3) mathematics concepts, mathematisation and reasoning; and (4) standard computational skills and carefulness in carrying out computations.

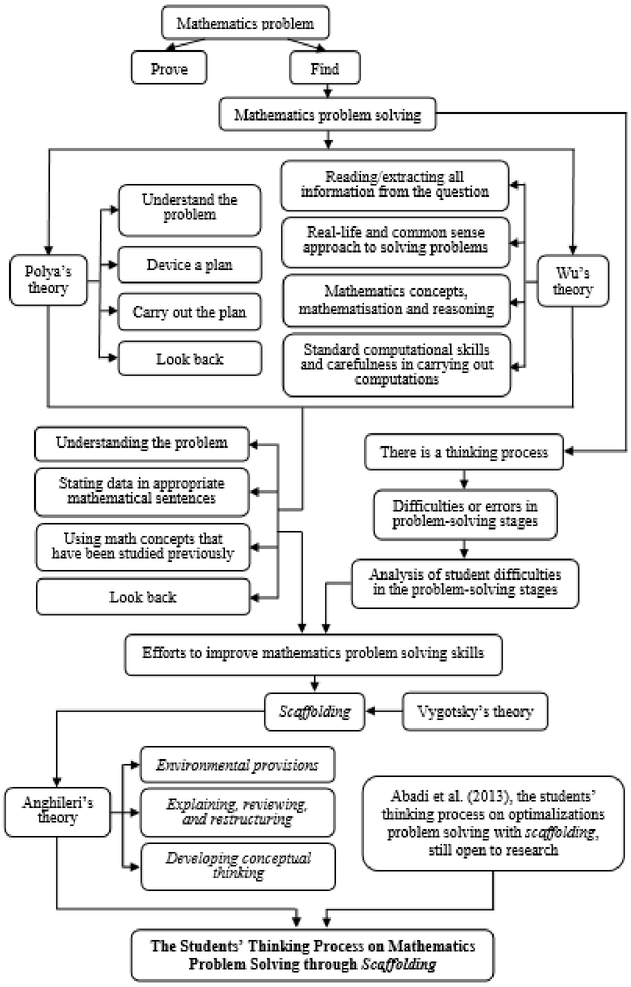
With reference to the four problem-solving stages proposed by Polya (1973) and Wu (2006), this research analyzes students'thinking process in every problem-solving stages that arises in the students' step in solving a mathematical problem. The problem-solving stages studied in this research are: (1) understanding the problem; (2) stating data/ information in appropriate mathematical sentences; (3) using and connecting mathematical concepts that have been studied previously; and (4) look back.

One form of efforts done to improve students' problem solving skills is by applying the social cognitive theory developed by Vygotsky. Vygotsky (in Slavin, 2011:58) states that a person will be able to solve problems that the level of difficulty is higher than his or her basic ability after he or she gets asistance from someone who is more capable (more competent). Such assistance may be instructions, encouragement, warnings, breaking down problems into solving stages or giving an example. Such assistance is referred to as *scaffolding*. The *scaffolding* used in this study refers to 3 levels of *scaffolding* that Anghileri (2006) has described: (1) environmental provisions; (2) explaining, reviewing, restructuring; And (3) developing conceptual thinking.

Generally in learning activities, *scaffolding* has often been done, even always happened in every process of learning mathematics in class. However, the *scaffolding* that has been done is unplanned, so there is no clear description of the students' mindset either before or after the *scaffolding*. The description of the students' mindset should be examined and can be used as a reference for improvement, planning and implementation of the next lesson. To get a clearer description of the problem experienced by the students, furthermore, the researcher analyze how the students' thinking process in solving simple problems related to the use of the concept of Congruence and Area of 2-Dimentional Geometric Shapes when they work individually or through *scaffolding*.

2 CONCEPTUAL FRAMEWORK

The conceptual framework in this research is presented in scheme 1 as follows.



Scheme 1. The conceptual framework

3 METHODS

This research is a descriptive research that uses qualitative approach. The research participants were 6 students of grade 9 on junior high school in Banyuwangi Regency, East Java, Indonesia. They were students who have learned the material of Congruence and 2-Dimentional Geometric Shape are. The participants are determined based on their level of mathematical ability as follows: 2 students with good mathematics ability; 2 students with medium mathematics ability; And 2 students with less mathematics ability. The determination of the mathematical ability of the students is based on the score of problem solving ability test, the mathematics value of the previous report, and the opinions from mathematics teacher and homeroom teacher. Determination of research participants also consider the smooth communication of students in expressing their ideas (to facilitate the process of *scaffolding*).

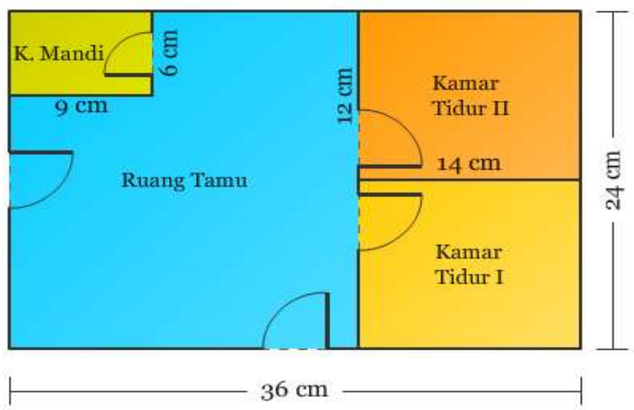
The data retrieval is done by giving one simple problem related to the material of congruence to be completed by some grade IX students. Students were asked to solve the problems given individually by writing down their steps clearly. Students who have been able to answer correctly all the problems, not used as research participants, otherwise students who have not been able to answer the problem properly were considered as the research participants. The researcher along with the Math teacher determined the participants of the research with the considerations as mentioned above. Students who have been assigned as the research participants were given the opportunity to do reflection on what he has done. Researcher conducted interview and invited them to discuss and guided them in order to improve the solution. When the participants fixed their solution, they were asked to express what they thought (Think Out Louds). The guidance of the researcher is intended to encourage the cognitive development of the participants so that they are able to solve the problems with higher difficulty levels. Furthermore, researchers observed and conducted an analysis of all activities performed by the participants.

4 RESULT AND DISCUSSION

In this research, the participants used consisted of three groups, namely subject of group A (two students with good math ability i.e S_1 and S_2), group B (two students with medium math ability i.e S_3 and S_4) and group C (two students with less math ability i.e S_5 and S_6), From each of these groups would be selected one as representative that would be discussed, they were namely S_2, S_4 and S_6. Description about participants' thinking process is presented before, during, and after *scaffolding*. The thinking structure of participants in solving mathematical problems before, during and after *scaffolding* is also illustrated, and then their thinking structures are compare to the structure of the given problem. The Description of thinking processes and their thinking structures are presented as follows.

4.1 *Participants' Thinking Process before Scaffolding*

The results show that participants' thinking process in mathematics problem solving given before *scaffolding* have not yet complete of the four stages of problem solving studied in this research. In average, participant has difficulty in using the mathematical concepts that have been studied previously and the stage of look back. In general, almost all participants have understood the basic formulas of congruence, but they have difficulties when they are exposed to mathematical problems in which the solutions require a combination of several previously learned math concepts. In addition, the difficulties they often encounter are in root concepts and in changing units. The given problem in this research is presented as follows.



Mr. Arif has a house with the sketch shown in the picture above. If the actual area of Mr. Arif's house is 54 m^2, determine:

a. The actual size of Mr. Arif's house!
b. The actual area of bedroom 1!

Furthermore, the problem structure in this research is presented in Figure 1.

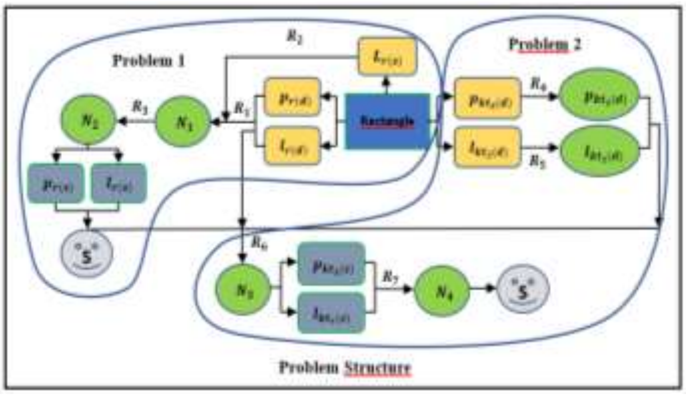
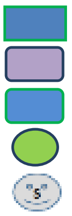


Figure 1. The structure of the problem

- Shape and colour description:



: Shape faced is rectangle.
: Data/information that has been known.
: Results of calculation for lenght and width.
: Result of calculation in general.
: The completion is done and correct.

- Code description:

Kode	*Meaning*
$L_{r(s)}$	*The actual area of the house*
$p_{r(d)}$	*The lenght of the house on the sketch*
$l_{r(d)}$	*The width of the house on the sketch*
$p_{kt_2(d)}$	*The lenght of bedroom 2 on the sketch*
$l_{kt_2(d)}$	*The width of bedroom 2 on the sketch*
R_1	*The rule of ration is* $p_{r(d)} : l_{r(d)}$
R_2	*The formula of rectangular area* $= p \times l$.
N_1	*Being able to specify the variable value of the length and width ratio of the house*
R_3	*Rule ratio of* R_1
N_2	*Being able to determine the actual size of the house*
R_4	$p_{kt_1(d)} = p_{kt_2(d)}$
R_5	$l_{kt_1(d)} = l_{r(d)} - l_{kt_2(d)}$
$p_{kt_1(d)}$	*The length of the bedroom 1 on the sketch*
$l_{kt_1(d)}$	*The width of bedroom 1 on the sketch*
R_6	*Congruence formula:* $\frac{p_{r(s)}}{p_{r(d)}} = \frac{p_{kt1(s)}}{p_{kt1(d)}}$ & $\frac{l_{r(s)}}{l_{r(d)}} = \frac{l_{kt1(s)}}{l_{kt1(d)}}$
N_3	*Being able to determine the actual size of bedroom 1*
R_7	*The formula of rectangular area* $= p \times l$.
N_4	*Being able to determine the actual area of bedroom 1*
S	*Completed and correct.*

The participant's thinking structure in solving the problems given before *scaffolding* compared with the structure of the given problem is presented in figures 2, 3 and 4.

a. *The thinking structure of S_2 (good math ability) before scaffolding.*

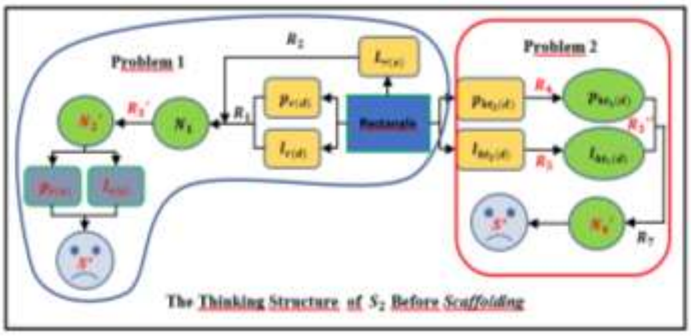


Figure 2. The thinking structure of S_2 before *scaffolding*.

Description:

Code	*Meaning*
$R_3{}'$	*The error in subtituting the actual size of the house.*
$N_2{}'$	*Being able to determine the actual size of the house, but not yet correct (because of dari $R_3{}'$).*
$R_3{}''$	*Using the same rule as $R_3{}'$, but not yet correct for the use of problem 2.*
$N_4{}'$	*Being able to determine the actual area of bedroom 1 but not yet correct.*
S'	*Completed but not yet true.*

b. *The thinking structure of S_4 (medium math ability) before scaffolding.*

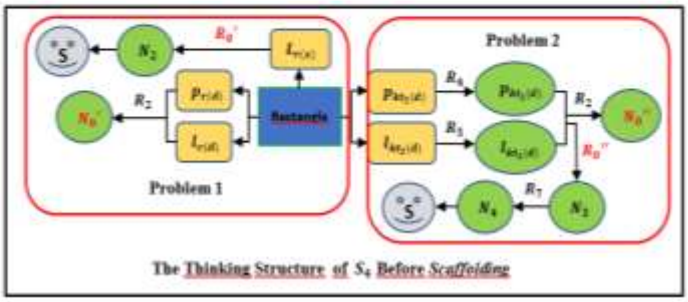


Figure 3. The thinking structure of S_4 before *scaffolding.*

Description:

Code	*Meaning*
$R_0{}'$	*Using logic to determine the actual size of the house (problem 1).*
$N_0{}'$	*Being able to determine the area of house on the sketch, but not using it to solve the problem 1.*

R_0''	*Using logic to determine the actual size of bedroom 1. (problem 2)*
N_0''	*Being able to determine the area of bedroom 1 on the sketch, but not using it to solve problem 2.*

c. *The thinking structure of S_6 (less math ability) before scaffolding.*

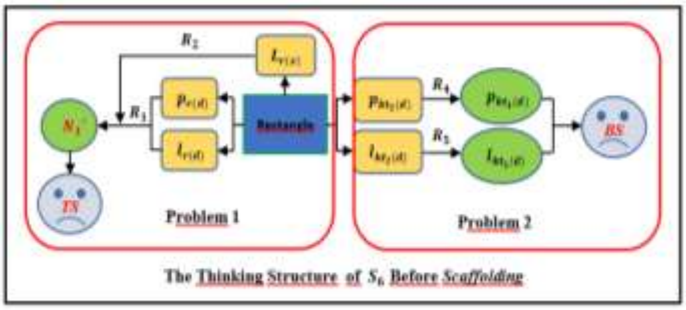


Figure 4. The thinking structure of S_6 before *scaffolding*.

Description:

Code	*Meaning*
N_1'	*Being unable to determine the value of x that is the variable value of the comparison of the length and width of the house.*
TS	*Not completed.*
BS	*Not yet at all.*

4.2 *Participants' Thinking Process during Scaffolding*

The participants' thinking process during *scaffolding* (while improving the given problem) has been so much developed so that the process has completed the four problem-solving stages examined in this research. By giving *scaffolding*, each participant (especially S_5 and S_6) can understand the purpose of the given problem, they have also been able to state the data/information they have acquired into appropriate mathematical sentences and complete them with images. In addition, they can also use mathematical concepts they have learned before and can connect those concepts to other concepts needed in solving the given problem. They were finally able to realize where is the mistakes they made before (before *scaffolding*). The participants' thinking structure in solving (fixing) the problems given during *scaffolding* is presented in Figures 5, 6 and 7.

a. *The thinking structure of S_2 (good math ability) during scaffolding.*

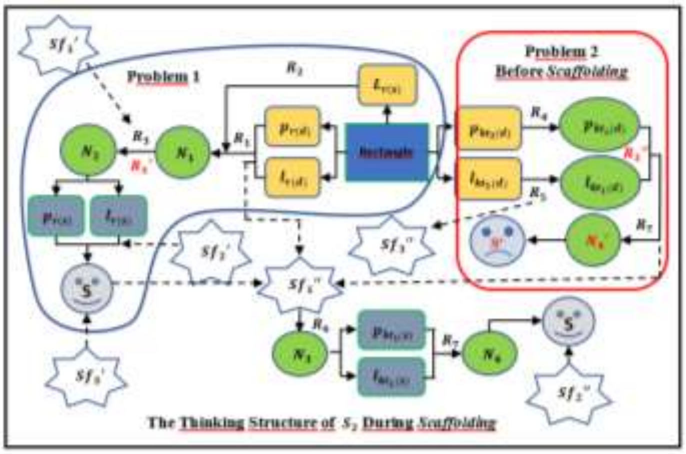
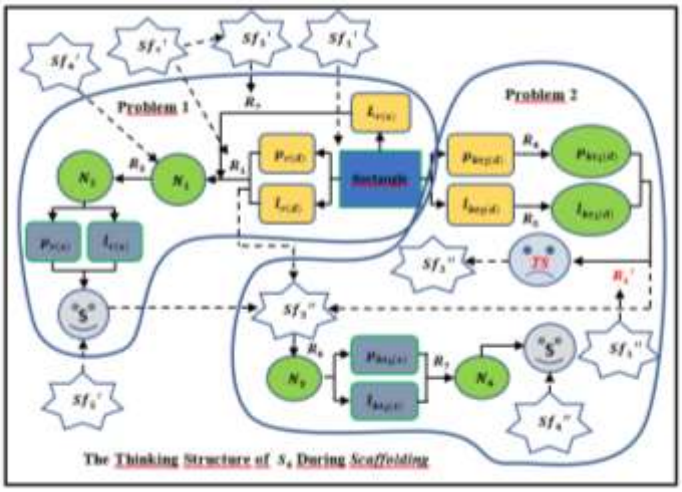


Figure 5. The thinking structure of S_2 during *scaffolding*

Description:

Code	*Meaning*
Sf_1'	*Scaffolding 1 (1): Looking, Touching and Verbalishing. Asking S_2 to do reflection on the result of his work and guide him to analize where is the mistake and correct the mistake.*
Sf_2'	*Scaffolding 2 (1): Prompting and Probing. Giving guidance to S_2 to improve the unit he has obbtained.*
Sf_3'	*Scaffolding 3 (1): Students Explaining and Justifying. Guiding S_2 to explain and justify the solution of problem 1.*
Sf_1''	*Scaffolding 1 (2): Looking, Touching and Verbalishing. Asking S_2 to do reflection on the result of his work and guide him to fix the mistake he has made.*
Sf_2''	*Scaffolding 2 (2): Students Explaining and Justifying. Guiding S_2 to explain and justify the solution of problem 2.*
Sf_3''	*Scaffolding 3 (2): Prompting and Probing. Guiding S_2 to review what is actually stated, so that later he will not make further mistake in the completion.*

b. *The thinking structure of S_4 (medium math ability) during scaffolding.*



Description:

Code	*Meaning*
Sf_1'	*Scaffolding 1 (1): Looking, Touching and Verbalishing. From Sf_1', it's known that S_4 has understood the meaning of the problem 1, and he has think about the concept of ratio (congruence) to solve problem 1.*
Sf_2'	*Scaffolding 2 (1): Prompting and Probing. Guiding S_4 to state the comparative value in the form of variable.*
Sf_3'	*Scaffolding 3 (1): Prompting and Probing. Giving leadning questions to S_4 so that he is able to use and connect the concept of congruence and the concept of 2-dimentional geometric shape area.*
Sf_4'	*Scaffolding 4 (1): Restructuring (restructing the comprehension): Providing meningful contexts. Writing down what the researcher meant on the papaer and then briefly explain to S_4 how to use and interconnect the concept of congruence and 2-dimentional geometric shape area.*
Sf_5'	*Scaffolding 5 (1): Students Explaining and Justifying. Guiding S_4 to explain and justify the solution of problem 1.*
Sf_1''	*Scaffolding 1 (2): Looking, Touching and Verbalishing. From Sf_1'', it's known that S_4 has understood the*

meaning of problem 2, but he is still confused about the what concepts that should be used to solve problem 2.

Sf_2''	*Scaffolding 2 (2): Prompting and Probing. Giving a few leading questions to S_4 so that he uses the concept of congruence in solving problem 2.*
Sf_3''	*Scaffolding 3 (2): Parallel Modelling. Giving relevant example related to the problem given.*
Sf_4''	*Scaffolding 4 (2): Students Explaining and Justifying. Guiding S_4 to be able to explain and justify the solution of problem 2.*

c. *The thinking structure of S_6 (less math ability) during scaffolding.*

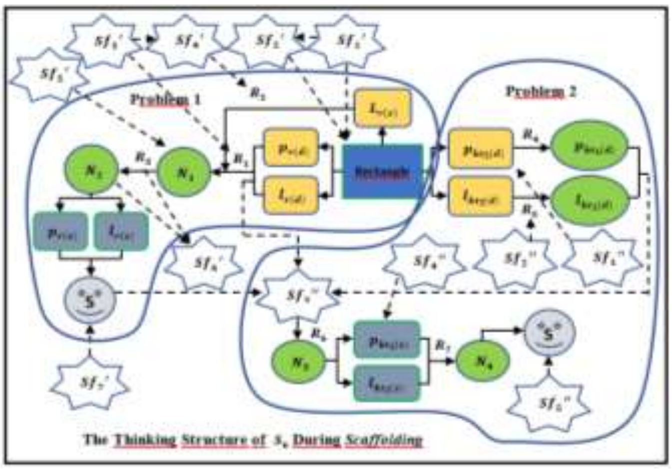


Figure 7. The thinking structure of S_6 during *scaffolding.*

Description:

Code	*Meaning*
Sf_1'	*Scaffolding 1 (1): Looking, Touching and Verbalishing. From Sf_1', it's knwon that S_6 has not understood the meaning of problem 1.*
Sf_2'	*Scaffolding 2 (1): Prompting and Probing. Guiding S_6 to comprehen the meaning of problem 1.*
Sf_3'	*Scaffolding 3 (1): Prompting and Probing. Guiding S_6 to use the consept of ratio and state them in the form of variable.*

Code	*Meaning*
Sf_4'	*Scaffolding 4 (1): Prompting and Probing. Giving some leading questions to S_6 so that he is able to review all the data in the problem.*
Sf_5'	*Scaffolding 5 (1): Prompting and Probing. Giving some leading questions to S_6 so that he is able to determine the value of x^2 and the value of x.*
Sf_6'	*Scaffolding 6 (1): Prompting and Probing. Giving a some leading questions to S_6 so that he is able to determine the expected units.*
Sf_7'	*Scaffolding 7 (1): Students Explaining and Justifying. Guiding S_6 to be able to explain and justify the solution of problem 1.*
Sf_1''	*Scaffolding 1 (2): Looking, Touching and Verbalishing. From Sf_1'', it's known that S_6 has understood the meaning of the problem 2, but he is not sure yet about the concept/formula he has used.*
Sf_2''	*Scaffolding 2 (2): Prompting and Probing. Guiding S_6 so that he is able to think about the first step he should do to solve problem 2.*
Sf_3''	*Scaffolding 3 (2): Parallel Modelling. Giving relevant example related the problem given.*
Sf_4''	*Scaffolding 4 (2): Prompting and Probing. Giving some leading questions to S_6 so that he is able to equate the units in doing the calculation.*
Sf_5''	*Scaffolding 5 (2): Students Explaining and Justifying. Guiding S_6 to be able to explain and justify the solution of problem 2.*

4.3 Participants' Thinking Process After Scaffolding

Participants' thinking process after *scaffolding* can be seen from the process and the addition problems solution and from their ability to find alternatives that are relevant to the all given problems. In solving addition problems, it appears that in general the participants' thinking process has complete all the four problem-solving steps studied in this research. They have been able to understand the problem without the need *scaffolding* again, they also have been able to solve the problem by utilizing the data/information they have gained from previous problems. In addition, they have also been able to use the right formula of congruence that has been taught from the previous problem. However, in general, the development of the participants' thinking process ceases at the stage of giving supplementary problems. On average, they (except S_2) have not been able to develop their ideas to find other relevant alternatives to all the problems given by using their mathematical reasoning. The additional problem in this research is presented as follows.

"From the previous problem, then determine the actual area of the bathroom 1!"

Furthermore, the structure of additional problem in this research is presented in Figure 8.

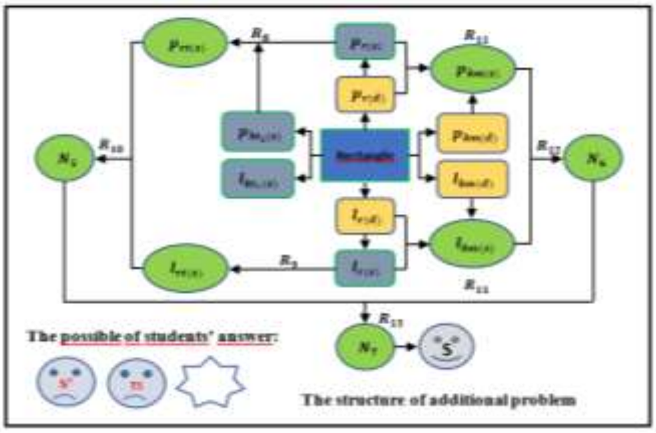


Figure 8. The structure of additional problem

Description:

Code	*Meaning*
$p_{r(d)}$ & $l_{r(d)}$	*The lenght and width of the house on the sketch*
$p_{r(s)}$ & $l_{r(s)}$	*The actual length and width of the house*
$p_{km(d)}, l_{km(d)}$	*The length and width of bathroom on the sketch*
$p_{kt_1(s)}, l_{kt_1(s)}$	*The actual length and width of the bathroom 1*
$p_{rt(s)}, l_{rt(s)}$	*The actual length and width of the living room*
R_8	$p_{rt(s)} = p_{r(s)} - p_{kt_1(s)}$
R_9	$l_{rt(s)} = l_{r(s)}$
R_{10}	*The formula of rectangle area*
N_5	*Being able to determine actual area of the living room (with the bath room)*
R_{11}	*Congruence formula* $\frac{p_{r(s)}}{p_{r(d)}} = \frac{p_{km(s)}}{p_{km(d)}}$ & $\frac{l_{r(s)}}{l_{r(d)}} = \frac{l_{km(s)}}{l_{km(d)}}$
$p_{km(s)}, l_{km(s)}$	*The actual length and width of the bathroom*
R_{12}	*The formula of the rectangle area*
N_6	*Being able to determine the actual area of the*

	bathroom
R_{13}	*The actual area of the living room (without bathroom)* $= L_{rt(s)} = L_{rt(s)'} - L_{km(s)}$
N_7	*Being able to determine the actual area of the living room (without bathroom)*
S	*Completed and correct*

Participants' thinking structure in solving the given problem after *scaffolding* (additional problems) compared with the problem structure, is presented in figures 9, 10 and 11.

a. *The thinking structure of* S_2 *(good math ability) after scaffolding.*

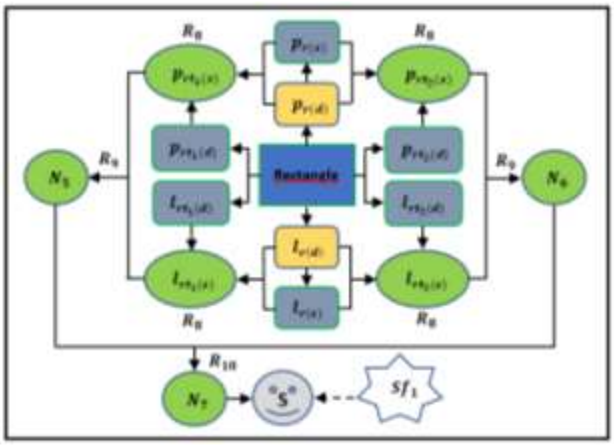


Figure 9. The thinking structure of S_2 after *scaffolding.*

Description:

Code	*Meaning*
Sf_1	*Scaffolding 1: Students Explaining and Justifying. Guiding* S_2 *to be able to explain and justify the solution.*

b. *The thinking structure of* S_4 *(medium math ability) after scaffolding.*

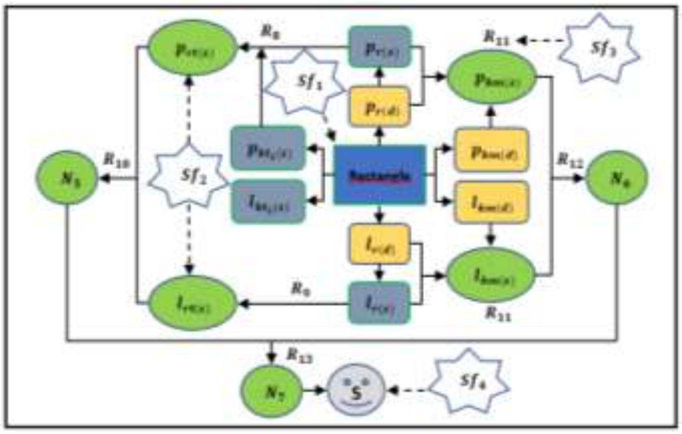


Figure 10. The thinking structure of S_4 after *scaffolding.*

Description:

Code	*Meaning*
Sf_1	*Scaffolding 1: Looking, Touching and Verbalishing and Restructuring. Guiding* S_4 *to be able to determine the shape of the living room that is rectangle and to be able to use the relevant way related to the previous problem in solving addition problem.*
Sf_2	*Scaffolding 2: Prompting and Probing. Giving a few leading questions to* S_4 *to be able to rechek the result of the calculation so that the mistake he does is known.*
Sf_3	*Scaffolding 3: Prompting and Probing. Guiding* S_4 *to be able to use another faser and easy alternative in determining the actual length and width of the living room that is by utilizing the obtained data from the previous problem.*
Sf_4	*Scaffolding 4: Students Explaining and Justifying. Guiding* S_4 *to be able to explain and justify the solution.*

c. *The thinking structure of* S_6 *(less math ability) after scaffolding.*

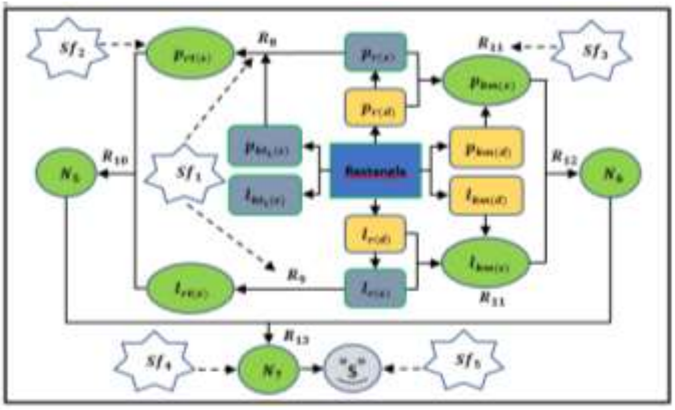


Figure 11. The thinking structure of S_6 after *scaffolding.*

Description:

Code	*Meaning*
Sf_1	*Scaffolding 1: Looking, Touching and Verbalishing and Prompting and Probing. Asking* S_6 *to look back at the data/information he has obtained from the previous problem, so he can use the data to determine the actual length and the width of the living room without using ratio.*
Sf_2	*Scaffolding 2: Prompting and Probing. Guiding* S_6 *untuk to be able to do addition or subtraction operation on decimal numbers.*
Sf_3	*Scaffolding 3: Restructuring (reconstructing comprehension). Asking* S_6 *to solve addition problem given using the same way he uses in solve the previous problem.*
Sf_4	*Scaffolding 4: Prompting and Probing. Guiding* S_6 *to be able to do addition and substraction operation on decimal numbers.*

Code	*Meaning*
Sf_5	*Scaffolding 5: Students Explaining and Justifying. Guiding* S_6 *to be able to explain and justify the solution of the problem.*

Based on the results of previous research, it appears that the development of thinking processes and the amount of *scaffolding* given to each participant differs, in which the diferences are influenced by the character of each participant (mathematical ability). The differences and the explanation of participants' thinking process in mathematics problem solving (adapted to the problem-solving stage studied in this research) before, during and after *scaffolding* are presented in Table 1.

Table 1. The Participants' Thinking Process through *Scaffolding*

Participant	Math ability	Participants' thinking process before scaffolding	Participants' thinking process during scaffolding	Participants' thinking process after scaffolding
S_2	Good	- Being able to solve problem 1 well, but careless in subtituting the value of x (not yet completed the 4 stages). - Unable to solve problem 2 well yet (using the same way on problem), because not yet accurate in reviewing all the data/information, both known data and data obtaine from problem 1 (not yet completed the 2 stages) and difficulty in applying congruence concept (not yet completed the 3 stages).	- Being able to complete his thinking process by fixing the solution toward problem 1 through 3 times scaffolding and 3 times scaffolding for problem 2.	- Being able to solve addition problem with once scaffolding. - Being able to develop the ideas to find and use the another relevant strategy or way related to the given problem obtained from mathematical reasoning, by giving 4 times scaffolding (problem 1) and 3 times (problem 2).
S_4	Medium	- Being able to solve problem 1 and 2 well and correctly, but only using logic in solving (not accompanien with the proper completion steps). It means that his thinking process has not yet completed the 3rd and the 4th stage both for problem 1 and 2.	- Being able to complete his thinking process by fixing the solution toward problem 1 through 5 times scaffolding and 4 times scaffolding for problem 2.	- Being able to solve addition problem by 4 times scaffolding. - Being able to develop the ideas to find another relevant strategy from problem 1 trough 5 times scaffolding (not yet finding the another relevant strategy or way related for problem 2).
S_6	Less	- Being unable to solve problem 1 and 2, because he has not understood the problem (problem 1 and 2) and find difficulty in in determining the formula that should be used in solving the given problem (being unable to use the concept of congruence yet). It means that his thinking process has not yet completed the 1st stage untill the 4th stage for either problem 1 or 2.	- Being able to complete his thinking process by fixing the solution toward problem 1 through 7 times scaffolding and 5 times scaffolding for problem 2.	ing able to solve addition problem trough 5 times scaffolding. - Being unable to develop the ideas to find another relevant strategy or way related of the given problem obtained from his mathematical reasoning.

Based on the discussion that has been described previously, it can be concluded that each participant thinking process through *scaffolding* improves a lot so that the thinking process completes the four stages of problem solving studied in this research. In general, with the *scaffolding* given by the researcher, each participant (especially S_5 and S_6) gradually understands the purpose of the problem (understanding the problem), they have also been able to state the data/information they have acquired into appropriate mathematical sentences and complete it with images. Not only that, they finally can also use the mathematical concepts they have learned before and can relate those concepts with other mathematical concepts needed to solve the problem (not only focus with 1 concept). By giving *scaffolding*, they can finally realize where is of the mistakes they made in solving the problem before (before *scaffolding*).

This finding is in line with some of the relevant research results that have been conducted by Abadi et al. (2013) and Hariana (2015). Previous research by Abadi et al. and Hariana point out that in general, giving the various *scaffolding* according to the needs of each participant can develop their thinking process. By giving *scaffolding* (Abadi and Hariana), participants can understand where is of the mistakes they have done and can improve the solution. In addition, by giving *scaffolding*, the ability of the participans in problem solving has increased.

In addition to the in line findings in some previous research results (research by Abadi and Hariana), it is also found that the most suitable *scaffolding* applied to all participants to complete the thinking process is *scaffolding* with the stage *Reviewing: Parallel Modeling*, it means that when the interaction that has been done is not enough to lead to the expected solution, alternative strategies that can be used is with the same modeling, i.e giving examples of similar or relevant problems and solutions that can be understood by the students. With the *scaffolding* of this stage, it appears that the participant can easily understand how to use the concept of congruence appropriately in solving the problems given. In addition, with this stage of *scaffolding*, the thinking processes of each participant progressed faster and better than the several stages of *scaffolding* given in this research.

4 CONCLUSION AND SUGGESTION

4.1 *Conclusion*

Based on the results of the research and discussion that has been described previously, it can be concluded that the participants' thinking process in mathematics problem solving through *scaffolding* can be described into three stages as follows.

a. Participants' thinking process before *scaffolding* all have not yet complete the four stages of problem solving studied in this research. In general, participants have difficulty in the stage of using mathematical concepts that have been studied previously. Most of them have understood the basic formula of congruence, but they have difficulty when they are exposed to a mathematical problem in which the solution requires some combination of concepts.

b. Participants' thinking process during *scaffolding* is on progress so that the thinking process complete all four problem-solving stages studied in this research. In addition, the structure of thinking has increased so that it matches the structure of the given problem. They are finally able to realize where is of the mistakes they made in solving the previous problem.

c. Participants' thinking process after *scaffolding* (additional problems) has generally completed the four problem-solving stages studied in this research. They have been able to understand the problem without the need for *scaffolding* again and can solve the problem well. However, the development of the participants' thinking process generally stops at this stage, almost all participants (except S_2) have not been able to develop his ideas to find other relevant solutions to all given problems.

The comparison of the participants thinking process development through *scaffolding* is described as follows.

a. The participants' thinking process with good mathematical ability is improved more rapidly than participants with medium and less math ability. In addition, *scaffolding* and time are given during the *scaffolding* process for participants with good math ability are fewer than participants with medium and less math ability.

b. The participant whose thinking process has the best and fastest development among other participants is S_2 (good math ability). He can fix the solution to a given problem by only 3 times *scaffolding*, he can also solve additional problems provided very well and smoothly with only 1 time *scaffolding*. Not only that, he can also develop his ideas to find and use other relevant strategies from all given problems.

c. The participants whose thinking process development take a long time and requires the most *scaffolding* among other participants is S_5 (less math ability). He can fix the solution to the problem with 9 times *scaffolding* (problem 1) and 7 times (problem 2). Besides, the additional problems given to S_5 differ from to other participants (similar to the previous problem). The reason is that he needs a lot of and repeat *scaffolding* so that his thinking process can develop. The development of his thinking process has not yet reached the stage of developing his ideas to obtain other strategies / other ways that are relevant to the given problem.

4.2 Suggestion

Based on the results of the research and the discussion that has been described previously, the researchers proposes some suggestions as follows.

a. Researchers in particular and teachers in general must be understand how far the thinking process and character of each student in mathematics problem solving, so that later teachers can develop a learning design that is able to facilitate all students in improving their mathematics problem solving skills.

b. *Scaffolding* given in this research is still limited, therefore the need for advanced research with more complex and structured *scaffolding*, to obtain a clearer illustration of students' mindset before and after the *scaffolding*, which further the students' mindset illustration can be used as one of the reference to make improvements, planning and implementation of the next lesson.

c. The problem given in this research is still simple there is involving only some mathematical concepts, therefore there is a need for further research with more in-depth study and with more complex problems.

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